ME 321: FLUID MECHANICS-I

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Lecture-08

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Fluid dynamics
- Linear Momentum Equation
- Bernoulli Equation

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Bernoulli Equation

The Bernoulli equation is a momentum-based force relation. It may be interpreted as an idealized energy relation.

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant (Bernoulli constant)}$$

Pressure head ($\frac{p}{\gamma}$)

Velocity head ($\frac{V^2}{2g}$)

Potential head (z)

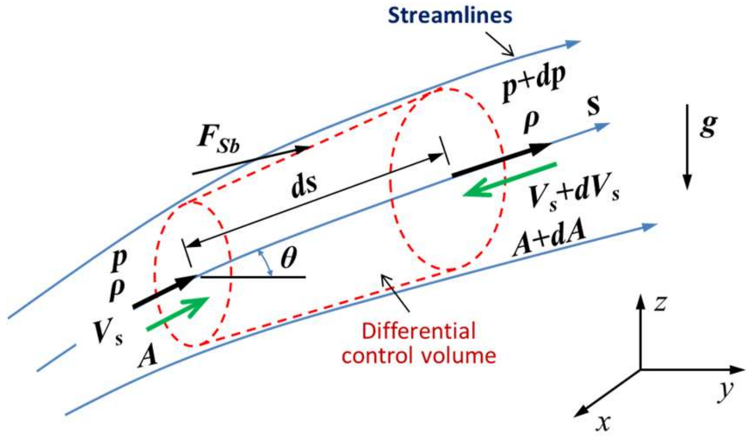


Fig. Differential control volume for momentum analysis of flow through a stream tube

Subjected to the following restrictions in fluid flow:

(i) Steady flow
(ii) Inviscid flow (no friction, ideal fluid flow, $\mu = 0$; no viscous dissipation)
(iii) Incompressible flow (density is constant or $M < 0.3$)
(iv) Irrotational flow (curl **V** = 0)
(v) Flow along a streamline

Differential Control Volume Analysis

Considering any two points in the flow field (irrotational flow), **Bernoulli Equation** yields:

$$\frac{p_1}{\gamma} + \frac{V_1^{\,2}}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^{\,2}}{2g} + z_2$$

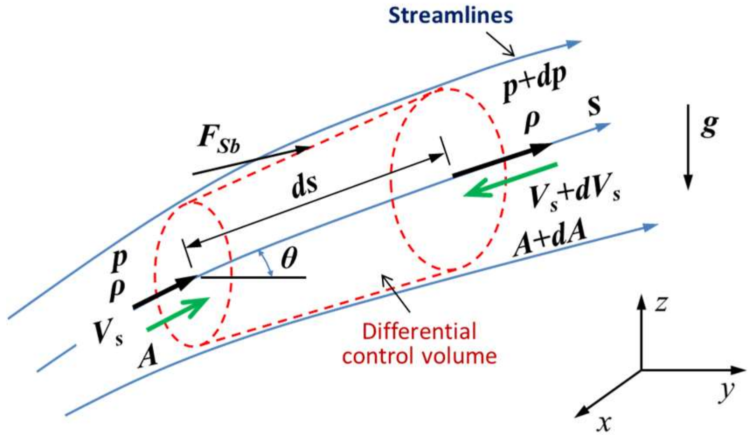


Fig. Differential control volume for momentum analysis of flow through a stream tube

Validity of Bernoulli equation

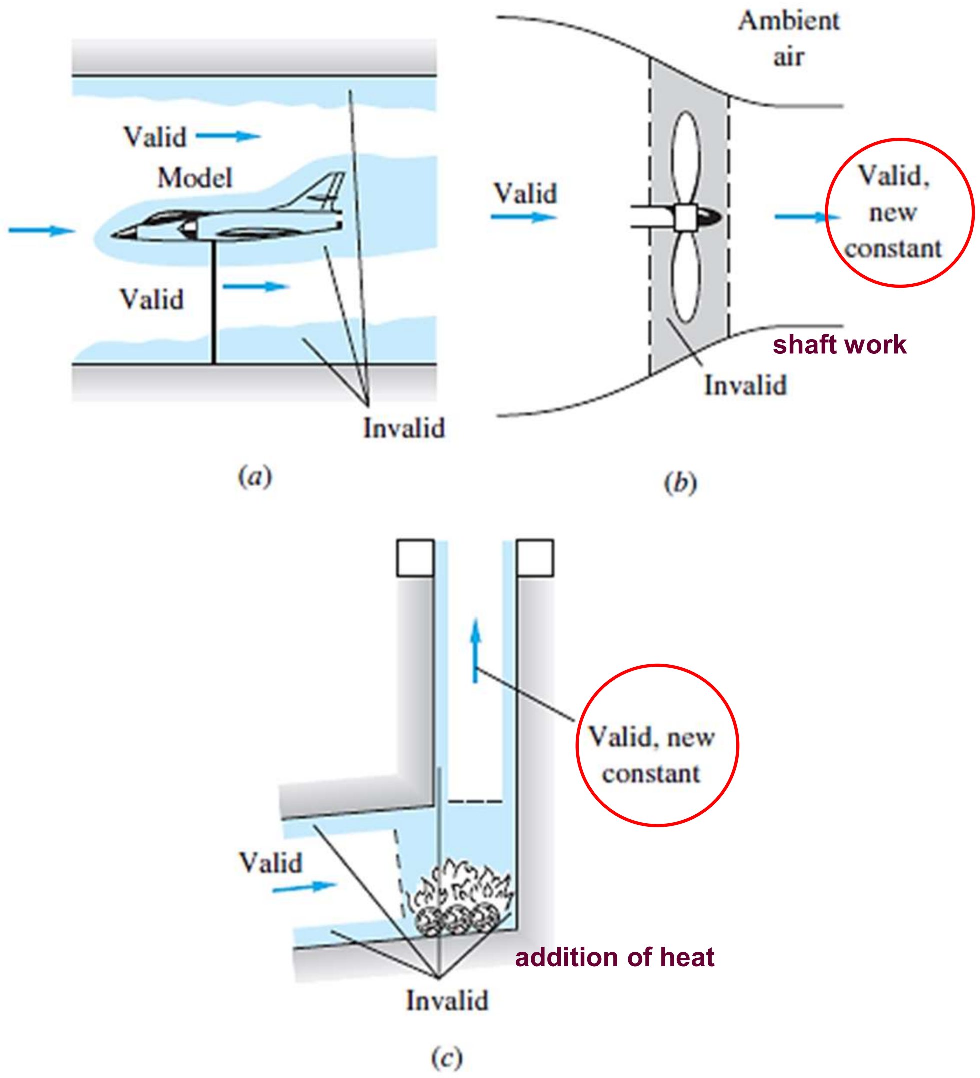
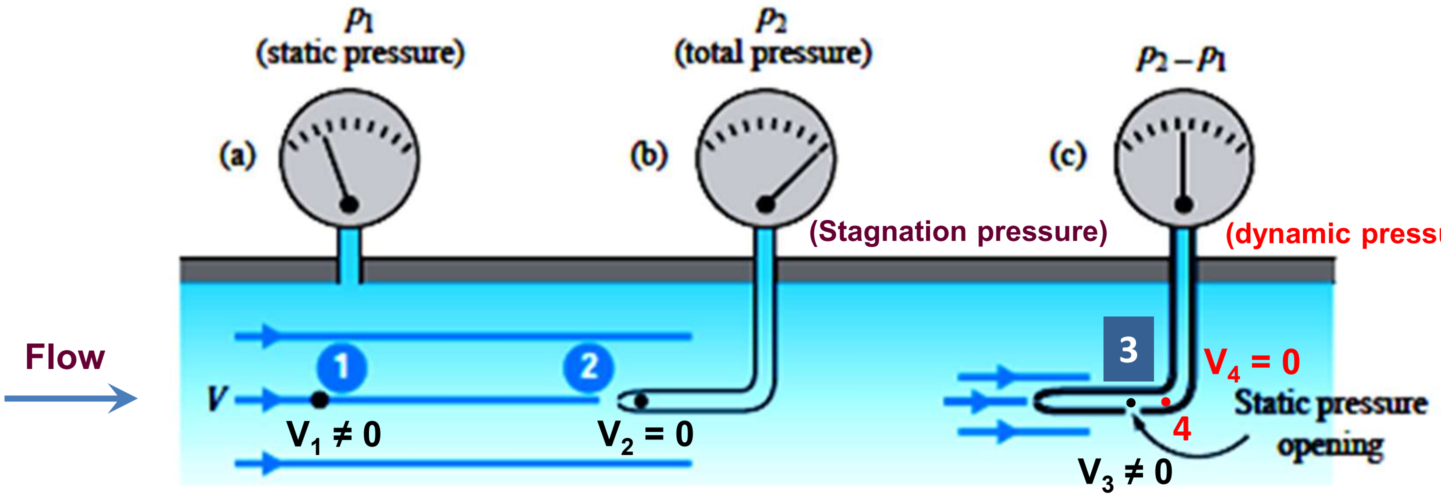


Fig. 3.13 Illustration of regions of validity and invalidity of the Bernoulli equation: (*a*) tunnel model, (*b*) propeller, (*c*) chimney.

Static, Dynamic & Stagnation Pressure



$$\frac{1}{2}\rho V^2$$

Fig. 3.18 Pressure probes: (a) piezometer; (b) pitot probe; (c) pitot-static probe.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$$\Rightarrow \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_0}{\gamma} + \frac{0^2}{2g}$$

$$\Rightarrow p_0 = p_1 + \frac{1}{2}\rho V_1^2$$

dynamic pressure: $\frac{1}{2}\rho V_1^2$

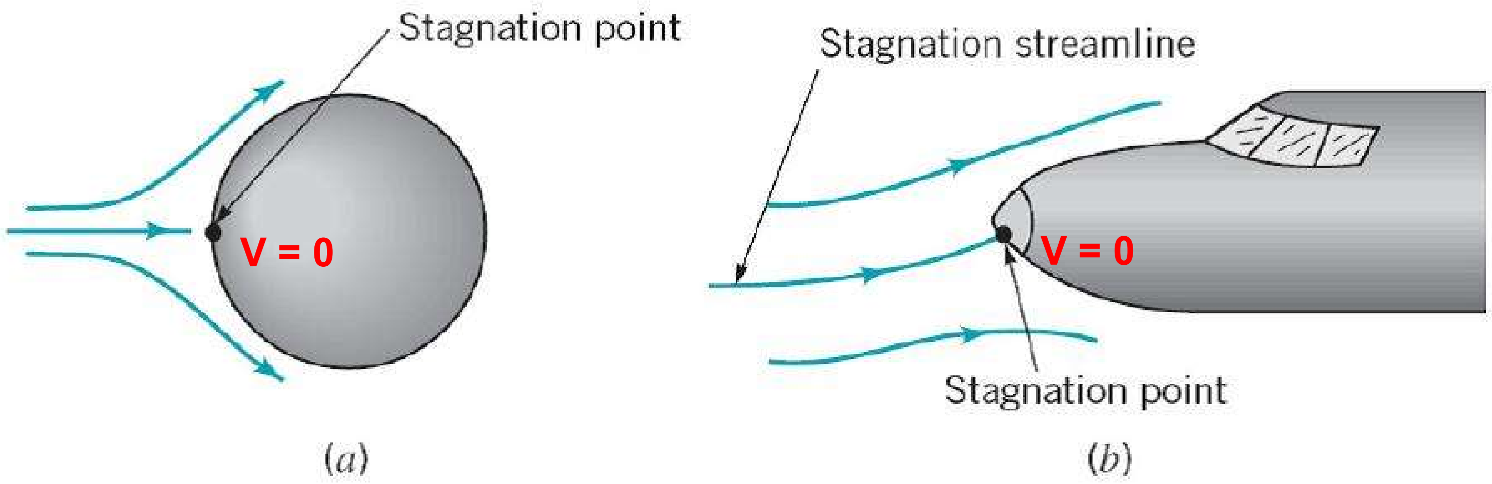
Stagnation/ total pressure: p_0

Static pressure: p_1

(Incompressible flow)

Stagnation Pressure = Static pressure + Dynamic Pressure

Stagnation Point



Stagnation points on bodies in flowing fluids.

(frictionless flow, no shaft work or heat transfer; EGL = const.)

EGL = Energy Grade Line

$$\left(\frac{p}{\gamma} + \frac{V^2}{2g} + z = h_0\right)$$

HGL = Hydraulic Grade Line

$$\left(\frac{p}{\gamma} + z\right)$$

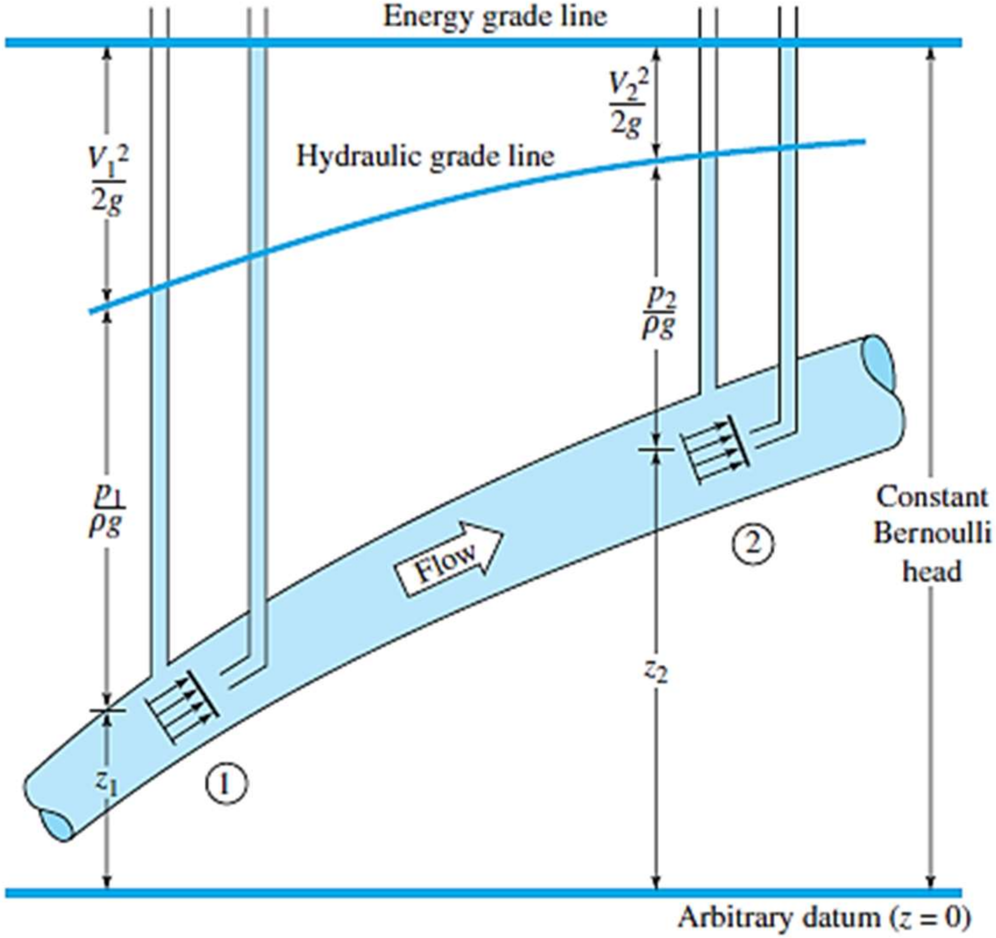
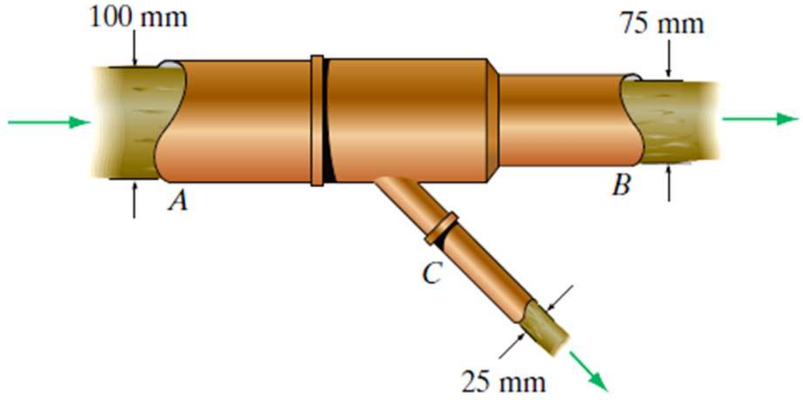


Fig. 3.14 Hydraulic and energy grade lines for frictionless flow in a duct.

Problem 1

Oil flows through the horizontal pipe under a pressure of 400 kPa and at a velocity of 2.5 m/s at A. Determine the pressure in the pipe B if the pressure at C is 150 kPa. Neglect any elevation difference. Take ρ = 880 kg/m^3



Solution:

From continuity equation, (steady flow)

$$\frac{d}{dt}\int_{CV} \rho d\mathcal{V} + \int_{CS} \rho\left(\vec{\mathbf{V}}\cdot\vec{\mathbf{n}}\right) dA = 0$$

$$\Rightarrow \int_{CS} \rho\left(\vec{\mathbf{V}}\cdot\vec{\mathbf{n}}\right) dA = 0$$

$$\Rightarrow -\rho A_A V_A + \rho A_B V_B + \rho A_C V_C = 0$$

$$\boxed{\Rightarrow A_A V_A = A_B V_B + A_C V_C} \qquad \textbf{(A)}$$

Not necessarily you have to start from the first relation but you have to realize that **Eq. (A)** is the result of the integral form of continuity equation for steady inviscid incompressible flows.

$$\Rightarrow \frac{\pi}{4} d_A^2 V_A = \frac{\pi}{4} d_B^2 V_B + \frac{\pi}{4} d_C^2 V_C$$

$$\Rightarrow \frac{\pi}{4}(0.1)^2(2.5) = \frac{\pi}{4}(0.075)^2 V_B + \frac{\pi}{4}(0.025)^2 V_C$$

$$\boxed{\Rightarrow 9V_B + V_C = 40 \qquad (i)}$$

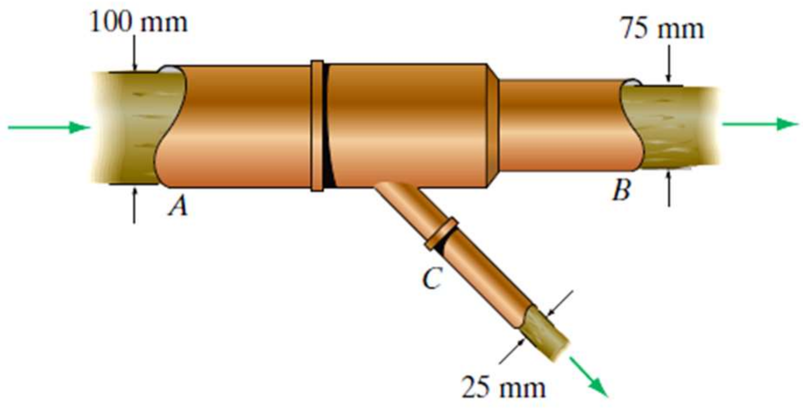
Problem 1

Apply Bernoulli equation between point A and C

$$\frac{p_A}{\gamma}+\frac{V_A^{\,2}}{2g}+z_A=\frac{p_C}{\gamma}+\frac{V_C^{\,2}}{2g}+z_C$$

$$\Rightarrow \frac{400\times10^3}{(880\times9.81)}+\frac{(2.5)^2}{2g}+\cancel{z_A}=\frac{150\times10^3}{(880\times9.81)}+\frac{V_C^2}{2g}+\cancel{z_C}$$

$$\boxed{\Rightarrow V_C=23.97\ \text{m/s}}$$



Now use Eq. (i)

$$9V_B+V_C=40 \qquad \rightarrow \qquad V_B=1.78\ \text{m/s}$$

Again apply Bernoulli equation between point A and B

$$\frac{p_A}{\gamma}+\frac{V_A^{\,2}}{2g}+z_A=\frac{p_B}{\gamma}+\frac{V_B^{\,2}}{2g}+z_B$$

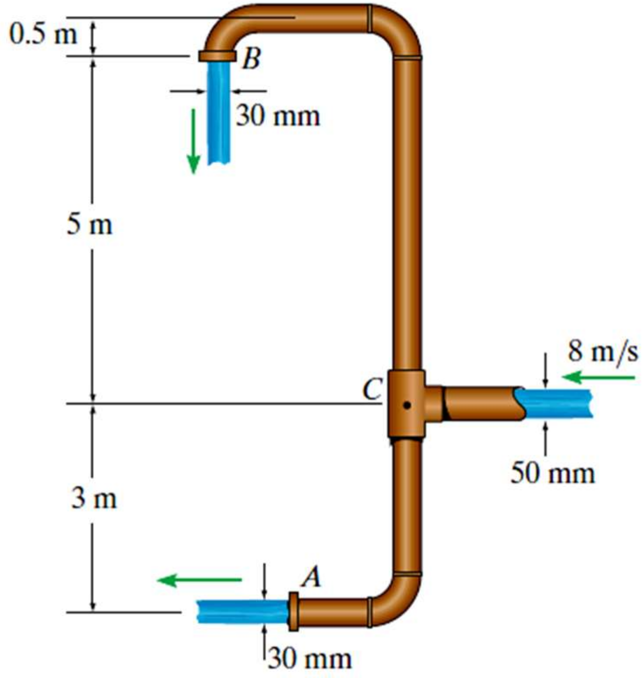
$$\Rightarrow \frac{400\times10^3}{(880\times9.81)}+\frac{(2.5)^2}{2g}+\cancel{z_A}=\frac{p_B}{(880\times9.81)}+\frac{(1.78)^2}{2g}+\cancel{z_C}$$

ratio of pressure to velocity heads??

$$\boxed{\Rightarrow p_B=401.4\ \text{kPa}}$$ **Ans.**

Problem 2

Determine the velocity of the flow out of the vertical pipes at A and B, if water flows into Tee at 8 m/s and under a pressure of 40 kPa.



Solution:

From continuity equation, (steady flow)

$$\frac{\partial}{\partial t}\int_{CV} \rho d\forall + \int_{CS} \rho\left(\vec{\mathbf{V}}\cdot\vec{\mathbf{n}}\right)dA = 0$$

$$\Rightarrow \int_{CS} \rho\left(\vec{\mathbf{V}}\cdot\vec{\mathbf{n}}\right)dA = 0$$

$$\Rightarrow -\rho A_C V_C + \rho A_A V_A + \rho A_B V_B = 0$$

Not necessarily you have to start from the first relation but you have to realize that **Eq** is the result of the integral form of continuity equation for steady incompressible flows.

$$\Rightarrow A_C V_C = A_A V_A + A_B V_B$$

$$\Rightarrow \frac{\pi}{4} d_C^2 V_C = \frac{\pi}{4} d_A^2 V_A + \frac{\pi}{4} d_B^2 V_B$$

$$\Rightarrow \frac{\pi}{4}(0.05)^2(8) = \frac{\pi}{4}(0.03)^2 V_A + \frac{\pi}{4}(0.03)^2 V_B$$

$$\Rightarrow V_A + V_B = 22.22 \qquad (i)$$

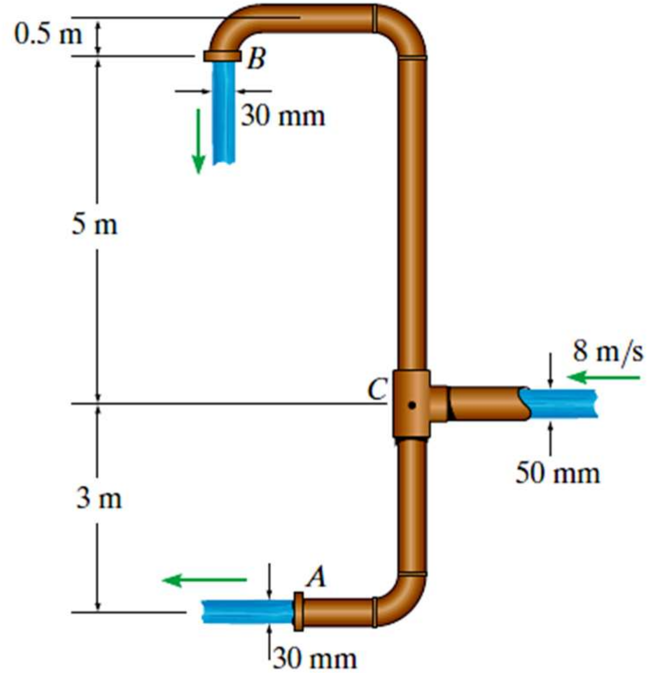
Problem 2

Apply Bernoulli equation between point C and A

$$\frac{p_C}{\gamma}+\frac{V_C^{\ 2}}{2g}+z_C=\frac{p_A}{\gamma}+\frac{V_A^{\ 2}}{2g}+z_A$$

$$\Rightarrow \frac{40\times10^3}{(1000\times9.81)}+\frac{(8)^2}{2g}+0=\frac{0}{(1000\times9.81)}+\frac{V_A^2}{2g}-3$$ $; p_A = p_B = 0$ (open discharge)

$$\Rightarrow V_A = 14.24 \text{ m/s}$$ **Ans.**

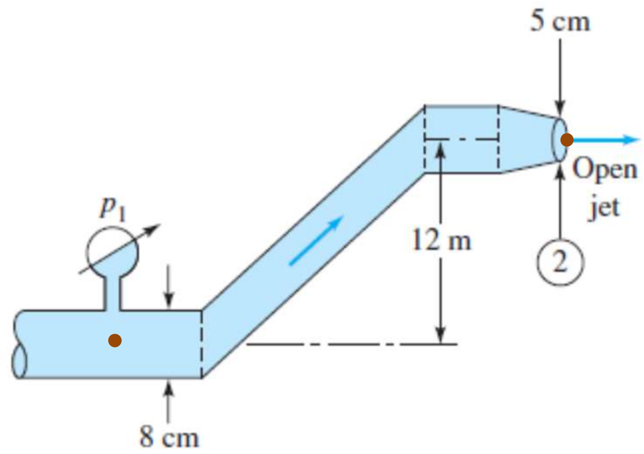


Now use Eq. (i)

$$V_A + V_B = 22.22 \quad \rightarrow \quad V_B = 7.98 \text{ m/s}$$ **Ans.**

Problem 3

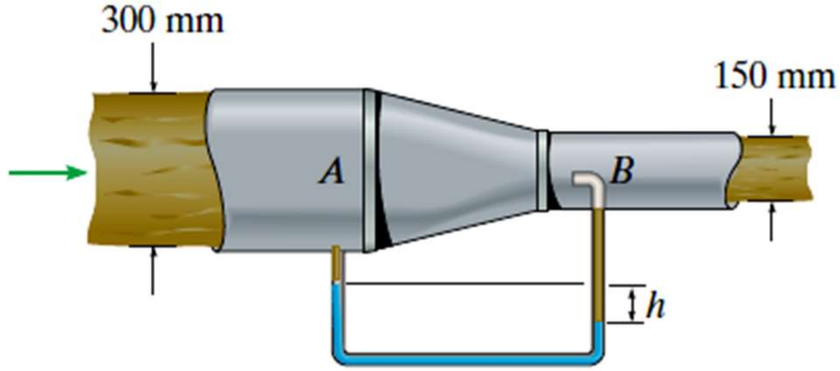
In Fig. below, the fluid is gasoline at 20°C (SG = 0.68). It flows at a weight flux of 120 N/s. Assuming no losses, estimate the gage pressure at section 1.



Ans: p_1 = 104.3 kPa

Problem 4

Determine the difference in height *h* of the water column in the manometer if the flow of oil through the pipe is 0.04 m³/s. Take ρ_{oil} = 875 kg/m³



Solution:

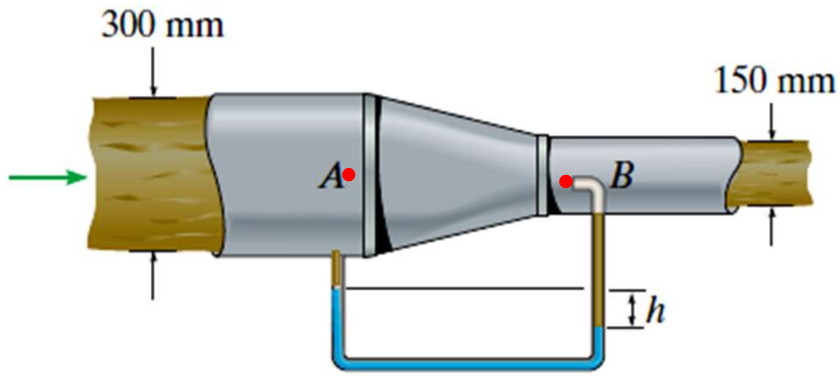
From continuity equation, (steady flow)

$$Q = A_A V_A = \ 0.04 \ \ \text{m}^3/\text{s} \qquad \text{(given)}$$

$$\therefore \ V_A = \frac{0.04}{\frac{\pi}{4} d_A{}^2} = \frac{0.04}{\frac{\pi}{4}(0.3)^2} = 0.57 \ \ \text{m/s}$$

$$B \ \text{ is the stagnation point} \quad \therefore \ V_B = 0$$

Problem 4



Apply Bernoulli equation between point A and B

$$\frac{p_A}{\gamma_{oil}} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma_{oil}} + \frac{V_B^2}{2g} + z_B$$

$$\Rightarrow \frac{p_A}{\gamma_{oil}} + \frac{(0.57)^2}{2g} + \cancel{z_A} = \frac{p_B}{\gamma_{oil}} + \frac{0^2}{2g} + \cancel{z_B}$$

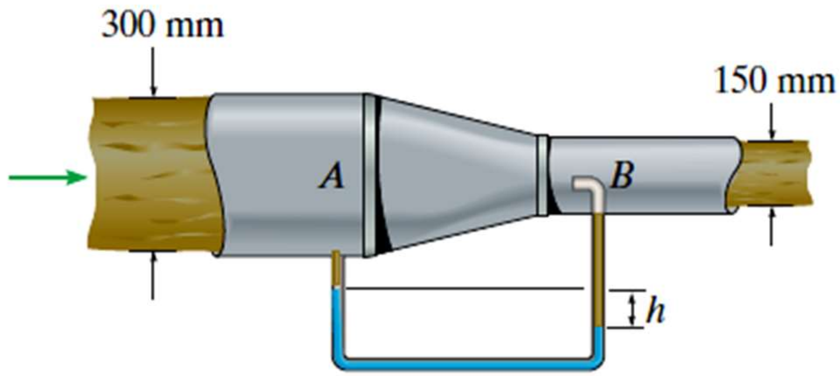
$$\Rightarrow p_B - p_A = \frac{(0.57)^2}{2g}\gamma_{oil}$$

$$\Rightarrow p_B - p_A = \frac{(0.57)^2}{2g}(\rho_{oil} g)$$

$$\Rightarrow p_B - p_A = \frac{(0.57)^2}{2}(875)$$

$$\boxed{\Rightarrow p_B - p_A = 142.1 \qquad (i)}$$

Problem 4



From **principle of manometry (fluid statics)**

$$p_A + \rho_{oil} g h_{AC} + \rho_{water} g h_{CD} = p_B + \rho_{oil} g h_{BD}$$

$$\Rightarrow p_A + (875)(9.81)a + (1000)(9.81)h = p_B + (875)(9.81)(a+h)$$

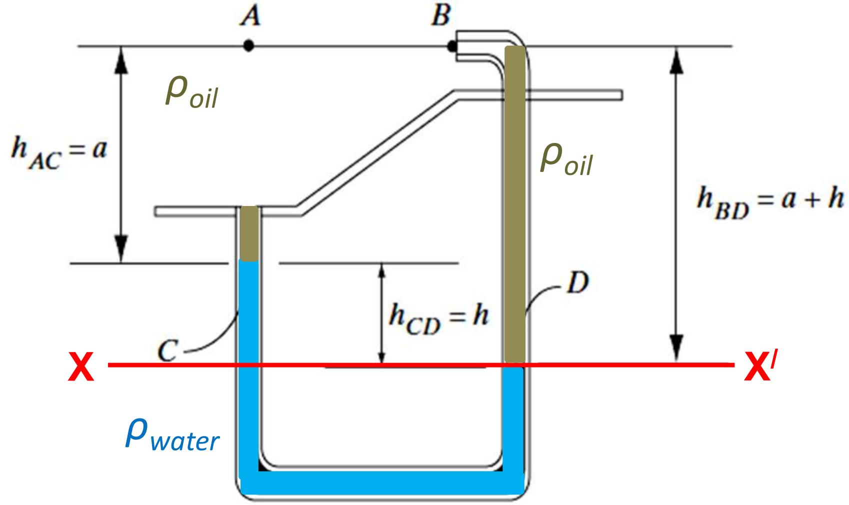
$$\Rightarrow p_B - p_A = 1226.25\,h$$

$$\Rightarrow 142.1 = 1226.25\,h$$

From Eq. (i): $\boxed{\Rightarrow p_B - p_A = 142.1}$

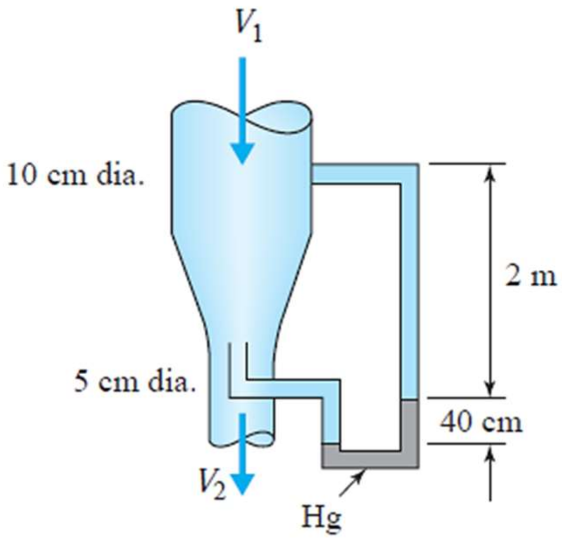
$$\boxed{\begin{array}{l} \Rightarrow h = 0.116\,\text{m} \\ \therefore h = 116\,\text{mm} \end{array}}$$

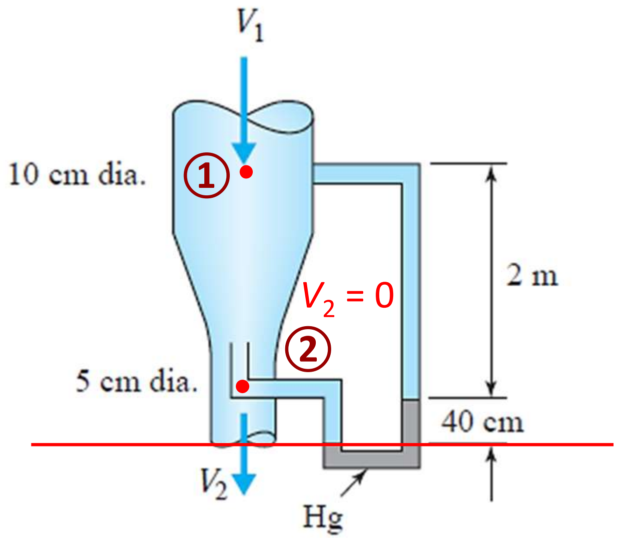
Ans.



Problem 5

Find the velocity V_1 of the water in the vertical pipe shown in Fig. Assume no losses.



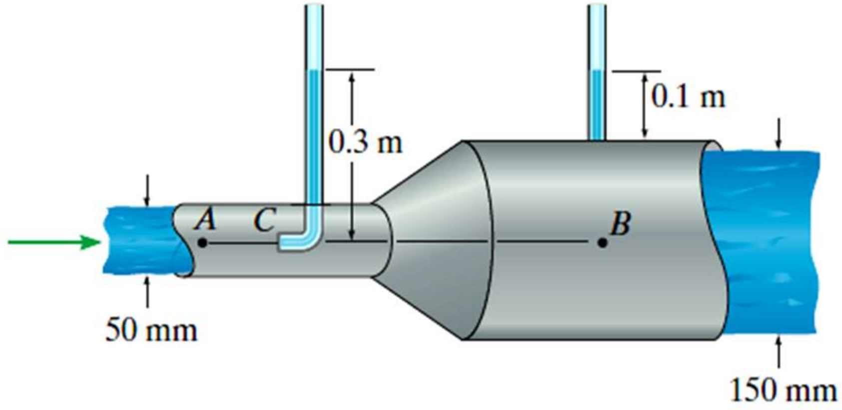


(1) Apply Bernoulli Equation between ① and ②
(2) Use the principle of manometry

Ans: V_1 = 9.94 m/s

Problem 6

Determine the volumetric flow rate of water and the pressure in the pipe at A if the height of the water column in the Pitot tube is 0.3 m and the height in the piezometer is 0.1 m.



Solution:

Use continuity equation between points A and B $V_A = 9V_B$

Calculate pressure at B $p_B = 1.71675\,(10^3)\,\text{Pa}$

Calculate stagnation pressure at C $p_C = 2.943\,(10^3)\,\text{Pa}$

Use Bernoulli equation between points **A** and **C** $p_A + 500V_A^2 = 2.943\,(10^3)$

Use Bernoulli equation between points **C** and **B** $V_B = 1.566\,\text{m/s}$

$$Q = A_A V_A = A_B V_B$$

Ans.

$$p_A = -96.4\,\text{kPa}$$

$$Q = 0.0277\,\text{m}^3/\text{s}$$

Real flow system

Modification of Bernoulli Equation is a must for **real flow system**:

Real flow system must account for loss of energy, which is frequently known as head loss.

1) **Major loss** (due to viscous effect /fluid friction /viscosity)
2) **Minor loss** (due to different pipe fittings, etc.)

Details in ME 323

Modified Bernoulli relation comes as:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L$$

(1) is the upstream point and
(2) is the downstream point

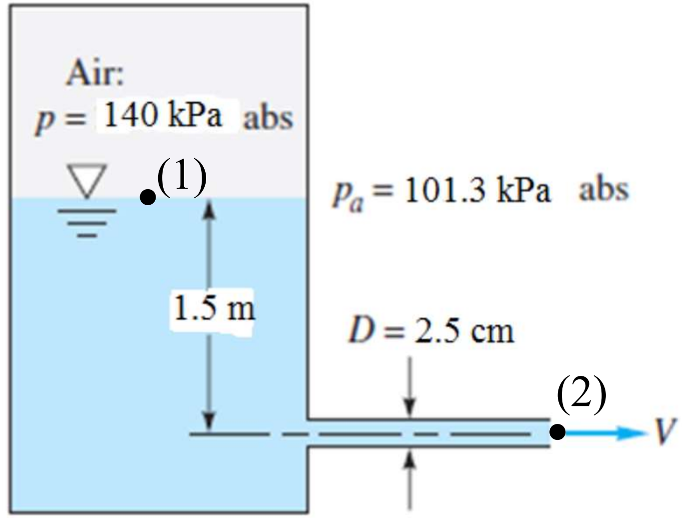
$$h_L = \text{head loss}$$

Problem 7

The liquid in the figure below is kerosene (SG = 0.8). Estimate the flow rate from the tank for

(a) No losses and

(b) Pipe losses $h_L \approx 4.5 \dfrac{V^2}{2g}$



Solution:

(a)
$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$

$$\Rightarrow \frac{140\times10^3}{\gamma}+\frac{0^2}{2g}+1.5=\frac{101.3\times10^3}{\gamma}+\frac{V_2^2}{2g}+0$$

$$\Rightarrow V_2 = \qquad (\equiv V)$$

$$\therefore\ Q = AV = ?$$

(b)
$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L$$

$$\Rightarrow \frac{140\times10^3}{\gamma}+\frac{0^2}{2g}+1.5=\frac{101.3\times10^3}{\gamma}+\frac{V_2^2}{2g}+0+4.5\frac{V_2^2}{2g}$$

$$\Rightarrow V_2 = \qquad (\equiv V)$$

$$\therefore\ Q = AV = ?$$